

Algorithms for the Asymmetric Traveling Salesman Problem

The traveling salesman problem is one of the most fundamental optimization problems. Given n cities and pairwise distances, it is the problem of finding a tour of minimum distance that visits each city once. In spite of significant research efforts, current techniques seem insufficient for settling the approximability of the traveling salesman problem. The gap in our understanding is especially large in the general asymmetric setting where the distance from city i to j is *not* assumed to equal the distance from j to i .

Indeed, it remains a notorious open problem to design an algorithm with *any* constant approximation guarantee. This is particularly intriguing as the standard linear programming relaxation is believed to give a constant factor approximation algorithm, where the constant may in fact be as small as 2.

In this talk, we will give an overview of old and new approaches for settling this question. We shall, in particular, talk about a new approach that shows that the integrality gap is a constant when restricted to shortest path metrics of node-weighted graphs. The main idea of our approach is to first consider an easier problem obtained by relaxing the general connectivity requirements into local connectivity conditions. For this relaxed problem, it is quite easy to give an algorithm with a guarantee of 3 on node-weighted shortest path metrics. More surprisingly, we then show that any algorithm (irrespective of the metric) for the relaxed problem can be turned into an algorithm for the asymmetric traveling salesman problem by only losing a small constant factor in the performance guarantee. This leaves open the intriguing task of designing a "good" algorithm for the relaxed problem on general metrics.